

## THE BODY WAVES DUE TO A GENERAL SEISMIC SOURCE IN A LAYERED EARTH MODEL: 1. FORMULATION OF THE THEORY

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### ABSTRACT

The radiation field exterior to any kind of volume source in a homogeneous medium can be represented in terms of an expansion in spherical harmonics. Such an expansion then provides an equivalent elastic source representation of quite general character in that nearly any proposed seismic source model, whether obtained using analytical or numerical (finite difference or finite element) methods, can be written in this form. The compatibility of this equivalent source with currently used source models, especially numerical models including detailed computations of the nonlinear processes at the source, is discussed. The equivalent source is then embedded in a stack of plane elastic layers representing the near-source crustal geology, and expressions are derived for computing the steeply emergent body waves exiting the base of the model. These displacements can then be combined with transfer functions representing the effect of the remainder of the travel path to compute theoretical seismograms for the body waves recorded in the far-field.

### INTRODUCTION

A fundamental objective of theoretical seismology is the development of computational methods for accurately simulating the propagation of seismic waves through the Earth. The last 10 to 15 years have seen intense activity aimed at the development of techniques for computing elastic waves in layered media. Widespread use is being made of such methods as generalized ray theory (e.g., Gilbert and Helmberger, 1972) and the reflectivity method (e.g., Fuchs and Müller, 1971) which can compute the far-field body waves in a spherically stratified elastic earth model to almost any desired precision.

Concurrent with improvements in wave propagation techniques, vastly improved methods for simulating the seismic source (earthquake or explosion) have also come into extensive use. These include complex analytical source theories such as that of Archambeau (1968) as well as the use of numerical finite difference/finite element methods. The numerical methods, particularly the finite difference formulations, are now being applied in attempts to directly simulate the nonlinear processes at the source (e.g., Cherry *et al.*, 1976b).

Much of the data through which we view the seismic source is recorded in the far-field and an important test of any source model is how well it agrees with the far-field ground-motion data. Therefore, it is necessary to have a bridge between the source calculations and the elastic-wave propagation techniques of theoretical seismology. That is, we need an equivalent elastic source.

An expansion of the outgoing elastic-wave field in spherical harmonics provides an equivalent elastic source of quite general character and nearly any seismic source model can be written in this form. Harkrider and Archambeau (1976) derived the expressions for computing surface waves for such a source embedded in a stack of plane elastic layers. In this paper the corresponding theory for the steeply emergent body waves exiting the base of this plane layered model is given. The derivation is closely related to

that of Fuchs (1966) and Hudson (1969a, b) who treated a similar problem except that their source was assumed to be given in terms of elementary point forces and their derivatives. Our results are, of course, completely equivalent where the source representations coincide.

In the following section the form of the equivalent elastic source and its compatibility with analytical and numerical source calculations is discussed. Subsequent sections contain the derivation of the far-field body waves for such a source in a layered elastic medium. Finally, we briefly discuss the computation of theoretical body-wave seismograms, using this formulation for the seismic source and crust in the source vicinity together with other methods for simulating the rest of the travel path. In a companion paper (Bache and Archambeau, 1976), the theoretical seismogram calculations are discussed in greater detail and a number of examples are given.

### EQUIVALENT ELASTIC SOURCE

The radiation field exterior to any kind of volume source in a homogeneous medium can be represented in terms of an expansion in spherical harmonics. Archambeau (1968) seems to have been the first to recognize the usefulness of this fundamental result and to apply it to geophysical problems. The expansion in spherical harmonics gives a compact equivalent elastic source representation of quite general character and nearly any proposed seismic source model can be cast in this form. A brief description of this source representation and its compatibility with commonly used source theories is the subject of this section.

The Fourier transformed equations of motion in a homogeneous, isotropic, linearly elastic medium may be written

$$\bar{\mathbf{u}} = - \left( \frac{1}{k_\alpha^2} \right) \nabla \bar{\chi}^{(4)} + \left( \frac{2}{k_\beta^2} \right) \nabla \times \bar{\chi}, \quad (1)$$

where  $\bar{\mathbf{u}}$  is particle displacement and  $k_\alpha$  and  $k_\beta$  are the compressional and shear-wave numbers. The Cartesian potentials  $\bar{\chi}^{(4)}$  and  $\bar{\chi}$  are defined by

$$\begin{aligned} \bar{\chi}^{(4)} &= \nabla \cdot \bar{\mathbf{u}}, \\ \bar{\chi} &= \frac{1}{2} \nabla \times \bar{\mathbf{u}}, \end{aligned} \quad (2)$$

and may be easily shown to satisfy the wave equation

$$\nabla^2 \bar{\chi}^{(j)} + k_i^2 \bar{\chi}^{(j)} = 0, \quad j = 1, 2, 3, 4, \quad (3)$$

where  $k_4 \equiv k_\alpha = \omega/\alpha$  and  $k_i \equiv k_\beta = \omega/\beta$  for  $i = 1, 2, 3$ . This equation has as a solution the following expansion in spherical eigenfunctions (e.g., Morse and Feshbach, 1953),

$$\begin{aligned} \bar{\chi}^{(j)}(\mathbf{R}, \omega) &= \sum_{l=0}^{\infty} h_l^{(2)}(k_j R) \sum_{m=0}^l [A_{lm}^{(j)}(\omega) \cos m\phi \\ &\quad + B_{lm}^{(j)}(\omega) \sin m\phi] P_l^m(\cos \theta), \end{aligned} \quad (4)$$

where the  $h_l^{(2)}$  are spherical Hankel functions of the second kind and the  $P_l^m$  are associated Legendre functions. The vector  $\mathbf{R}$  has as components the spherical coordinates  $R, \theta, \phi$ .

Equations (4), together with (1), provide an elastic point source representation of the (outgoing) displacement field. The values of the multipole coefficients,  $A_{lm}^{(j)}(\omega)$ ,  $B_{lm}^{(j)}(\omega)$ ,  $j = 1, 2, 3, 4$ , prescribe the displacement field at all points in the homogeneous medium

where (1) applies. This point source representation can be viewed as a generalized form for a sum of a monopole or center of dilatation ( $l = 0$ ), a dipole or couple ( $l = 1$ ), a quadrupole or double-couple ( $l = 2$ ), etc. For example, a center of dilatation is represented by a single coefficient  $A_{00}^{(4)}$ , while for a horizontal double couple the nonzero coefficients are  $-A_{21}^{(1)} = B_{21}^{(2)} = A_{22}^{(3)}$  and  $B_{22}^{(4)}$ .

Description of the character of the elastic field generated by seismic sources is, of course, a basic geophysical problem. For this paper it is convenient to discuss seismic source descriptions in three categories:

1. Those obtained using finite difference/finite element numerical methods.
2. Analytical source models of relaxation type.
3. Dislocation source models.

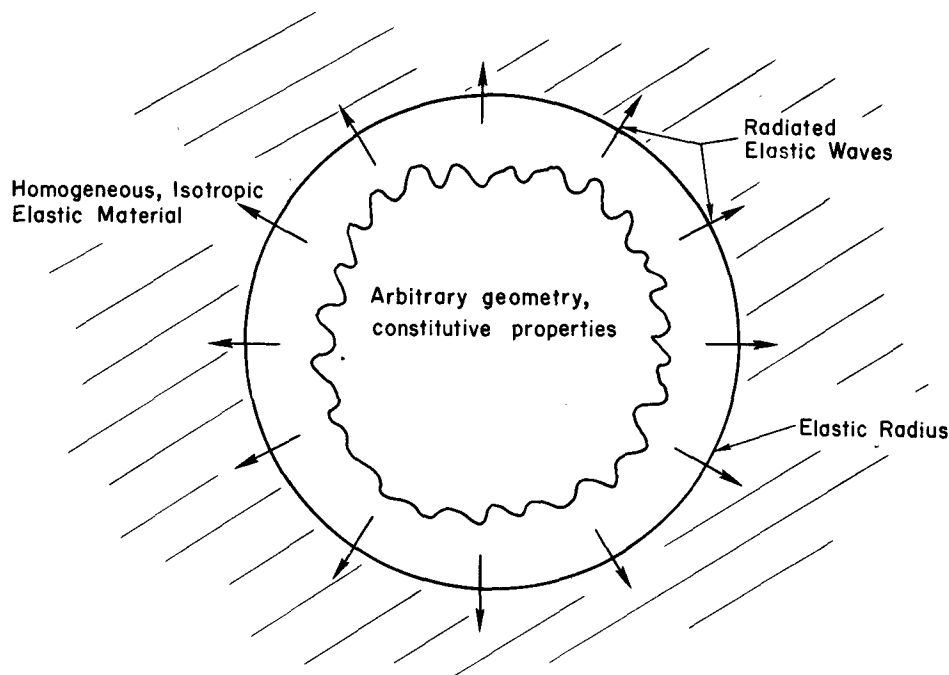


FIG. 1. Schematic display of the determination of an equivalent elastic representation for an arbitrary volume source.

With numerical methods one can attempt to directly include complexities of the source mechanism in a deterministic computational scheme. For example, finite difference methods have been extensively used to compute the propagating shock wave due to an underground nuclear explosion (e.g., Cherry *et al.*, 1974). In this case the nonlinear behavior of the rock under high stress loading determines the character of the seismic signal. If the source region can be assumed to be embedded in a medium in which (1) applies, an equivalent elastic source of the form (4) can be obtained from the outgoing displacement field. This is indicated schematically in Figure 1. Briefly, the procedure is to monitor the outgoing displacement field or, alternatively, the potentials,  $\tilde{\chi}^{(j)}$ , on a spherical surface of radius  $\bar{R}$ . Using the orthogonality of the spherical harmonics, these potentials are related to the multipole coefficients by

$$\begin{Bmatrix} A_{lm}^{(j)}(\omega) \\ B_{lm}^{(j)}(\omega) \end{Bmatrix} = \frac{C_{lm}}{h_l^{(2)}(k_j \bar{R})} \int_0^{2\pi} \int_0^\pi \tilde{\chi}^{(j)}(\bar{R}, \omega) P_l^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \sin \theta d\theta d\phi, \quad (5)$$

where

$$C_{lm} = \frac{(2l+1)(l-m)!}{2\pi(l+m)}, \quad m \neq 0,$$

$$C_{l0} = (2l+1)/4\pi.$$

Use of this procedure for linking nonlinear finite difference source calculations with analytical wave propagation techniques was suggested to the first author by Archambeau (1973, personal communication), and has since been implemented for a number of complex explosion geometries (Cherry *et al.*, 1975, 1976a), and for a three-dimensional finite difference simulation of stick-slip earthquake faulting (Cherry *et al.*, 1976b). The number of terms required for the expansion (4) to converge depends on the symmetry of the source radiation at frequencies of interest. The most elementary application of the method is for one-dimensional (spherically symmetric) explosion source calculations. For such problems the elastic field is often described by a reduced displacement potential defined by

$$U(R, t) = \frac{\partial}{\partial R} \left[ \frac{\Psi(t - R/\alpha)}{R} \right]. \quad (6)$$

Applying the Fourier transform and comparing to (1) together with (4), it is easily derived that

$$A_{00}^{(4)}(\omega) = -ik_\alpha^3 \bar{\Psi}(\omega), \quad (7)$$

which shows the equivalence between the reduced displacement potential and the monopole. For more complex sources such as an explosion in an axisymmetric tunnel (Cherry *et al.*, 1975) or several explosions detonated simultaneously (Cherry, *et al.*, 1976a) quadrupole and higher-order terms occur in the expansion. When an earthquake source is computed, the leading term is, as expected, the quadrupole (Cherry *et al.*, 1976b).

Three-dimensional relaxation models of the seismic source mechanism have been developed by Archambeau (1968, 1972) and Minster (1973). These authors present their results in terms of an expansion in spherical harmonics. One form of Archambeau's model has been used in a number of studies of induced tectonic stress release by underground nuclear explosions (Archambeau and Sammis, 1970; Archambeau, 1972; Bache, 1976). In this symmetric problem only the quadrupole term occurs in the expansion. When the Archambeau/Minster model is used to represent propagating ruptures (earthquakes), many terms may be required for the expansion (4) to converge, depending on fault length, rupture velocity and frequency.

The most common approach to earthquake source theory is to assume that dislocation theory is applicable (e.g., Haskell, 1964; Savage, 1966). Such theories generally represent the source in terms of a double-couple with frequency content depending on the dislocation time history assumed. Harkrider (1976) has given the expressions relating dislocation source theories to the expansion in spherical harmonics (4). For a horizontal double-couple (normal strike-slip fault) Harkrider (1976), equation (36), gives the Cartesian potentials as

$$\bar{\phi} = -i \frac{\mu \bar{D}(\omega)}{4\pi\rho\omega^2} k_\alpha^3 \sin 2\phi \frac{P_2^2(\cos \theta)}{3} h_2^{(2)}(k_\alpha R),$$

$$\bar{\psi}_1 = -i \frac{\mu \bar{D}(\omega)}{4\pi\rho\omega^2} k_\beta^3 \cos \phi \frac{P_2^1(\cos \theta)}{3} h_2^{(2)}(k_\beta R),$$

$$\begin{aligned}\bar{\psi}_2 &= i \frac{\mu \bar{D}(\omega)}{4\pi\rho\omega^2} k_\beta^3 \sin \phi \frac{P_2^{(1)}(\cos \theta)}{3} h_2^{(2)}(k_\beta R), \\ \bar{\psi}_3 &= i \frac{\mu \bar{D}(\omega)}{4\pi\rho\omega^2} k_\beta^3 \cos 2\phi \frac{P_2^{(2)}(\cos \theta)}{3} h_2^{(2)}(k_\beta R),\end{aligned}\quad (8)$$

where  $\rho$  and  $\mu$  are the density and shear modulus, and  $\bar{D}(\omega)$  represents the Fourier transformed dislocation time history. The implied dimensions of these potentials are per unit fault area. The results can be generalized to finite faults by introducing an integration over the fault surface. For fault models on which the dislocation history is invariant and the variation of phase between dislocation points and observer is negligible over the fault plane, (8) multiplied by the fault area are the potentials for a normal strike-slip fault.

The Cartesian potentials ( $\bar{\phi}$ ,  $\bar{\psi}$ ) are related to the displacements by

$$\bar{\mathbf{u}} = \nabla \bar{\phi} + \nabla \times \bar{\psi}. \quad (9)$$

Comparing to (1), we see that

$$\begin{aligned}\bar{\chi}^{(4)} &= -k_\alpha^2 \bar{\phi}, \\ \bar{\mathbf{x}} &= \frac{k_\beta^2}{2} \bar{\psi}.\end{aligned}\quad (10)$$

Using (10) and comparing (9) to (4), the multipole coefficients for the normal strike-slip dislocation are

$$\begin{aligned}B_{22}^{(4)}(\omega) &= \frac{i\mu \bar{D}(\omega)k_\alpha^3}{12\pi(\lambda + 2\mu)}, \\ A_{21}^{(1)}(\omega) &= \frac{-i\bar{D}(\omega)k_\beta^3}{24\pi}, \\ B_{21}^{(2)}(\omega) &= A_{22}^{(3)}(\omega) = -A_{21}^{(1)}(\omega).\end{aligned}\quad (11)$$

Expressions from which the multipole coefficients for a double couple at arbitrary orientation can be obtained are given by Harkrider (1976), Appendix A.

Whether the seismic source is modeled by numerical methods or analytical theories, the multipole coefficients provide a computationally convenient equivalent elastic source. Thus far, the discussion has been restricted to the multipolar expansion with respect to a coordinate system fixed with respect to the source. Minster (1973) gives the transformation matrices by which the multipole coefficients in a standard coordinate system (e.g., fixed with respect to the surface of the Earth) may be obtained from any rotated system. Using these results, multipole coefficients can be computed in a convenient source-related system and then rotated to a fixed geographical system.

In the following section the equivalent elastic source in the form discussed here will be embedded in a multilayered medium and we will derive the expressions for computing the steeply emergent, far-field body waves.

#### THE EQUIVALENT ELASTIC SOURCE IN A MULTILAYERED HALF-SPACE

A number of authors have investigated the elastic waves radiating from point sources in a multilayered medium overlying a homogeneous half-space. Most of these studies have relied on Thomson-Haskell matrix theory (Thomson, 1950; Haskell, 1953) as will

our derivation. Harkrider (1964) developed solutions for surface waves due to elementary point forces at depth. Fuchs (1966) derived transfer functions which include the effect of the layered crustal model on the far-field  $P$  waves from three types of sources: a center of dilatation, a couple and a double couple. Hudson (1969a, b) extended Fuchs' body-wave results to apply to quite general sources of finite extent and derived the analogous theory for surface waves. However, in all these theories the source representation is in terms of elementary point forces and their derivatives. The representation in this form of complicated sources (including terms of higher order than the double couple) at arbitrary orientation would appear to be an arduous task. Thus, the usefulness of these previous results, especially for routine numerical computations, seems to be limited by the inherent complexity of the source representation.

The multipolar expansion discussed in the previous section provides a unique, compact and convenient numerical representation for seismic sources of arbitrary complexity and orientation. Harkrider and Archambeau (1976) have computed the surface waves for a source given in this form embedded in a multilayered medium. This required the formulation of the displacement field in terms of integrals over wavenumber,  $k$ . The surface waves are then given by the residue contribution to these integrals. For the body waves it is necessary to evaluate the branch line contribution to similar integrals and this is the main result presented here. Derivation of the  $k$  integrals in the appropriate form follows closely the derivations of Harkrider (1964) and Harkrider and Archambeau (1976) and the notation is, for the most part, the same. It should be pointed out that our results are completely equivalent to those of Hudson (1969a, b). The difference is, of course, in the source representation.

Consider a semi-infinite elastic medium made up of  $n$  parallel, homogeneous, isotropic elastic layers. Number the layers from 1 at the free surface to  $n$  for the underlying half-space. Place the origin of a cylindrical coordinate system  $(r, \phi, z)$  at the free surface and denote the layer interfaces by  $z_i$ ,  $i = 1, 2, \dots, n-1$ . This geometry is depicted in Figure 2. Let  $(\bar{q}_i, \bar{v}_i, \bar{w}_i)$  be the components of the Fourier transformed displacements in the  $(r, \phi, z)$  directions in the  $i$ th layer. Then, following Harkrider (1964), the cylindrical potentials  $\bar{\Phi}_i, \bar{\Psi}_i, \bar{\Omega}_i$  are defined by

$$\begin{aligned}\bar{q}_i(r, \phi, z) &= \frac{\partial \bar{\Phi}_i}{\partial r} + \frac{\partial^2 \bar{\Psi}_i}{\partial r \partial z} + \frac{1}{r} \frac{\partial \bar{\Omega}_i}{\partial \phi}, \\ \bar{v}_i(r, \phi, z) &= \frac{1}{r} \frac{\partial \bar{\Phi}_i}{\partial \phi} + \frac{1}{r} \frac{\partial^2 \bar{\Psi}_i}{\partial z \partial \phi} - \frac{\partial \bar{\Omega}_i}{\partial r}, \\ \bar{w}_i(r, \phi, z) &= \frac{\partial \bar{\Phi}_i}{\partial z} + \frac{\partial^2 \bar{\Psi}_i}{\partial z^2} + k_{\beta i}^2 \bar{\Psi}_i, \quad i = 1, 2, \dots, n. \quad (12)\end{aligned}$$

We will subsequently be interested in an equivalent elastic point source at a depth  $z = h$ . Let the source layer be denoted by a subscript  $s$ ; that is,  $z_s < h < z_{s-1}$ . It is necessary to express the cylindrical potentials  $(\bar{\Phi}_s, \bar{\Psi}_s, \bar{\Omega}_s)$  in the source layer in terms of the spherical potentials,  $\bar{\chi}^{(j)}$ , from (4). The equivalence is given by Harkrider and Archambeau (1976) and is as follows

$$\begin{aligned}\bar{\Phi}_s &= \sum_{m=0}^l \int_0^\infty \{\bar{A}_m \cos m\phi + \bar{B}_m \sin m\phi\} [\exp(-ikr_\alpha|z-h|)/r_\alpha] J_m(kr) dk, \\ \bar{\Psi}_s &= \sum_{m=0}^l \int_0^\infty \{\bar{E}_m \cos m\phi + \bar{F}_m \sin m\phi\} [\exp(-ikr_\beta|z-h|)/r_\beta] J_m(kr) dk,\end{aligned}$$

$$\bar{\Omega}_s = \sum_{m=0}^l \int_0^\infty \{ \bar{C}_m^{(3)} \cos m\phi + \bar{D}_m^{(3)} \sin m\phi \} \exp(-ikr_\beta |z-h|/r_\beta) [J_m(kr)/k^2] dk, \quad (13)$$

where

$$\begin{aligned} \bar{A}_m &= -\frac{1}{k_\alpha^3} \sum_{l=m}^\infty (i)^{m-n} [\operatorname{sgn}(h-z)]^{m+l} A_{lm}^{(4)} (-1)^{m+n} P_l^m \left( \frac{kr_\alpha}{k_\alpha} \right), \\ \bar{B}_m &= -\frac{1}{k_\alpha^3} \sum_{l=m}^\infty (i)^{m-n} [\operatorname{sgn}(h-z)]^{m+l} B_{lm}^{(4)} (-1)^{m+n} P_l^m \left( \frac{kr_\alpha}{k_\alpha} \right), \\ \bar{C}_m^{(j)} &= \frac{2}{k_\beta^3} \sum_{l=m}^\infty (i)^{m-n} [\operatorname{sgn}(h-z)]^{m+l} A_{lm}^{(j)} (-1)^{m+n} P_l^m \left( \frac{kr_\beta}{k_\beta} \right), \\ \bar{D}_m^{(j)} &= \frac{2}{k_\beta^3} \sum_{l=m}^\infty (i)^{m-n} [\operatorname{sgn}(h-z)]^{m+l} B_{lm}^{(j)} (-1)^{m+n} P_l^m \left( \frac{kr_\beta}{k_\beta} \right), \end{aligned} \quad (14)$$

where  $j = 1, 2, 3$  in the definitions of  $\bar{C}_m^{(j)}$ ,  $\bar{D}_m^{(j)}$ , and

$$\begin{aligned} k &= \omega/c, \quad k_\gamma = \omega/\gamma, \\ r_\gamma &= (c^2/\gamma^2 - 1)^{1/2}, \quad \gamma = \alpha \text{ or } \beta, \\ c &= \text{horizontal phase velocity.} \end{aligned} \quad (15)$$

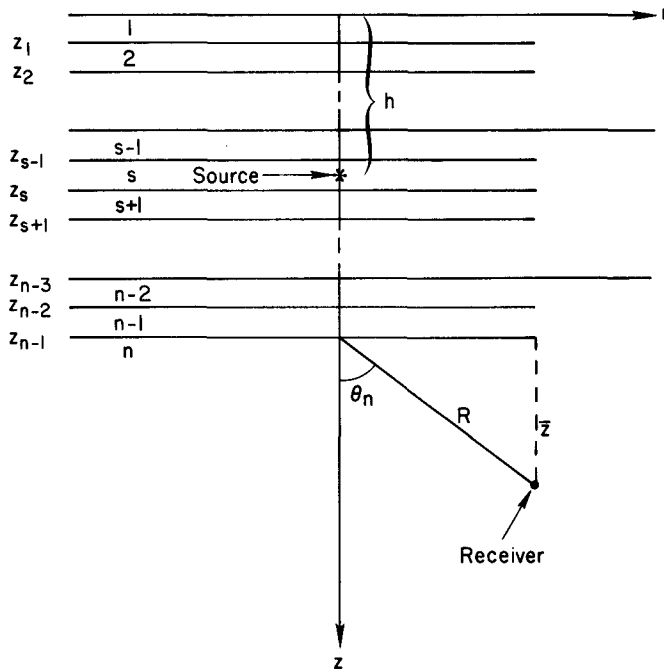


FIG. 2. The geometry and coordinate system for a source at depth  $h$  in a multilayered half-space.

The coefficients  $\bar{E}_m, \bar{F}_m$  in the expression for  $\bar{\psi}_s$  are related to the  $\bar{C}_m^{(j)}, \bar{D}_m^{(j)}$  by

$$\begin{aligned} 2k\bar{E}_0 &= \bar{C}_1^{(2)} - \bar{D}_1^{(1)}, \quad \bar{F}_0 = 0, \\ 2k\bar{E}_1 &= \bar{C}_2^{(2)} - \bar{D}_2^{(1)} - 2\bar{C}_0^{(2)}, \\ 2k\bar{F}_1 &= \bar{C}_2^{(1)} + \bar{D}_2^{(2)} + 2\bar{C}_0^{(1)}, \end{aligned} \quad (16)$$

and, taking  $\bar{C}_m^{(j)} = D_m^{(j)} = 0$  for  $m > l$ ,

$$\begin{aligned} 2k\bar{E}_m &= \bar{C}_{m+1}^{(2)} - \bar{C}_{m-1}^{(2)} - \bar{D}_{m+1}^{(1)} - \bar{D}_{m-1}^{(1)}, \\ 2k\bar{F}_m &= \bar{C}_{m+1}^{(1)} + \bar{C}_{m-1}^{(1)} + \bar{D}_{m+1}^{(2)} - \bar{D}_{m-1}^{(2)}, \end{aligned} \quad 2 \leq m \leq l.$$

Since the location of material boundaries depends only on  $z$ , the dependence on  $r$  and  $\phi$  will be everywhere the same as in the source layer. Therefore, separate the potentials in the layers as follows

$$\begin{Bmatrix} \bar{\Phi}_i(r, \phi, z) \\ \bar{\Psi}_i(r, \phi, z) \\ \bar{\Omega}_i(r, \phi, z) \end{Bmatrix} = \sum_{m=0}^l \int_0^\infty \begin{Bmatrix} \Phi_i^{(m)}(\phi, z) \\ \psi_i^{(m)}(\phi, z) \\ \Omega_i^{(m)}(\phi, z) \end{Bmatrix} J_m(kr) dk, \quad (17)$$

$i = 1, 2, \dots, n.$

The potentials  $\bar{\Phi}_i^{(m)}$ ,  $\psi_i^{(m)}$ ,  $\Omega_i^{(m)}$  satisfy wave equations for which the general solutions may be written

$$\begin{aligned} \Phi_i^{(m)}(\phi, z) &= \tilde{\Delta}_i^{(m)} \exp(-ikr_{\alpha_i}z) + \tilde{\Delta}_i^{(m)'} \exp(ikr_{\alpha_i}z), \\ \psi_i^{(m)}(\phi, z) &= \tilde{\omega}_i^{(m)} \exp(-ikr_{\beta_i}z) + \tilde{\omega}_i^{(m)'} \exp(ikr_{\beta_i}z), \\ \Omega_i^{(m)}(\phi, z) &= \tilde{\epsilon}_i^{(m)} \exp(-ikr_{\beta_i}z) + \tilde{\epsilon}_i^{(m)'} \exp(ikr_{\beta_i}z). \end{aligned} \quad (18)$$

In (18) and subsequent equations subscripts  $i$ ,  $i = 1, 2, \dots, n$ , denote quantities in the  $i$ th layer.

Then, define

$$\begin{aligned} \hat{\Delta}_i^{(m)} &= -k^2 \left( \frac{c}{\alpha_i} \right)^2 \exp(-kr_{\alpha_i}z_{i-1}) \hat{\Delta}_i^{(m)}, \\ \hat{\omega}_i^{(m)} &= \frac{ik^3}{\gamma_i} \exp(-ikr_{\beta_i}z_{i-1}) \tilde{\omega}_i^{(m)}, \\ \hat{\epsilon}_i^{(m)} &= k \exp(-ikr_{\beta_i}z_{i-1}) \tilde{\epsilon}_i^{(m)}. \end{aligned} \quad (19)$$

We will subsequently be interested in the values of the potential in the half-space ( $i = n$ ). Applying the radiation condition at infinity, (18) and (19) give

$$\begin{aligned} \Phi_n^{(m)}(\phi, z) &= -\frac{1}{k_{\alpha_n}^2} \exp(-ikr_{\alpha_n}\bar{z}) \hat{\Delta}_n^{(m)}, \\ \psi_n^{(m)}(\phi, z) &= -\frac{i\gamma_n}{k^3} \exp(-ikr_{\beta_n}\bar{z}) \hat{\omega}_n^{(m)}, \\ \Omega_n^{(m)}(\phi, z) &= \frac{1}{k} \exp(-ikr_{\beta_n}\bar{z}) \hat{\epsilon}_n^{(m)}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} \bar{z} &= z - z_{n-1}, \\ \gamma_i &= 2\beta_i^2/c^2. \end{aligned} \quad (21)$$

Now, combining (17) and (20), we have

$$\bar{\Phi}_n(r, \phi, z) = \sum_{m=0}^l \int_0^\infty \left( -\frac{1}{k_{\alpha_n}^2} \right) \hat{\Delta}_n^{(m)} \exp(-ikr_{\alpha_n}\bar{z}) J_m(kr) dk,$$



$$\begin{aligned}\bar{\psi}_n(r, \phi, z) &= \sum_{m=0}^l \int_0^\infty -\frac{i}{k} \left( \frac{2}{k_\beta^2} \right) \hat{\omega}_n^{(m)} \exp(-ikr_{\beta n} \bar{z}) J_m(kr) dk, \\ \bar{\Omega}_n(r, \phi, z) &= \sum_{m=0}^l \int_0^\infty \frac{1}{k} \hat{\epsilon}_n^{(m)} \exp(-ikr_{\beta n} \bar{z}) J_m(kr) dk.\end{aligned}\quad (22)$$

In equations (22) we now have the cylindrical potentials  $\bar{\Phi}_n$ ,  $\bar{\psi}_n$ ,  $\bar{\Omega}_n$ , in the  $n$ th layer (half-space) in terms of a sum of integrals of the Sommerfeld type. The coefficients  $\hat{\Delta}_n^{(m)}$ ,  $\hat{\omega}_n^{(m)}$ ,  $\hat{\epsilon}_n^{(m)}$  depend on azimuth,  $\phi$ , as well as wave number,  $k$ , and include the modification of the source generated pulse by the material discontinuities in the layered half-space. Following Harkrider (1964, equations 62 and 122), these coefficients are solutions of the matrix equations

$$\begin{aligned}\begin{bmatrix} \hat{\Delta}_n^{(m)} \\ \hat{\Delta}_n^{(m)} \\ \hat{\omega}_n^{(m)} \\ \hat{\omega}_n^{(m)} \end{bmatrix} &= J^R \left\{ \begin{bmatrix} \dot{u}_{R1}(0)/c \\ \dot{w}_{R1}(0)/c \\ 0 \\ 0 \end{bmatrix} + A_{RS1}^{-1} \begin{bmatrix} \delta U_m \\ \delta W_m \\ \delta Z_m \\ \delta X_m \end{bmatrix} \right\}, \\ \begin{bmatrix} \hat{\epsilon}_n^{(m)} \\ \hat{\epsilon}_n^{(m)} \end{bmatrix} &= J^L \left\{ \begin{bmatrix} \dot{v}_{L1}(0)/c \\ 0 \end{bmatrix} + A_{LS1}^{-1} \begin{bmatrix} \delta V_m \\ \delta Y_m \end{bmatrix} \right\}\end{aligned}\quad (23)$$

where the  $J^R$  and  $J^L$  are given by equations (61) and (124) of Harkrider (1964). The matrix  $A_{RS1}$  is defined by Harkrider and is the layer product matrix which gives the displacement-stress vector for  $P$ - $SV$  motion at the source depth in terms of the displacement-stress vector at the surface. Similarly,  $A_{LS1}$  is the transfer matrix for the displacement-stress vector associated with  $SH$  motion.

The source terms in (23) are given by Harkrider and Archambeau (1976) as follows

$$\begin{aligned}\delta U_m &= \delta \left( \frac{\dot{u}_s}{c} \right)_m^c \cos m\phi + \delta \left( \frac{\dot{u}_s}{c} \right)_m^s \sin m\phi, \\ \delta W_m &= \delta \left( \frac{\dot{w}_s}{c} \right)_m^c \cos m\phi + \delta \left( \frac{\dot{w}_s}{c} \right)_m^s \sin m\phi, \\ \delta Z_m &= \delta \sigma_m^c \cos m\phi + \delta \sigma_m^s \sin m\phi, \\ \delta X_m &= \delta \tau_{Rm}^c \cos m\phi + \delta \tau_{Rm}^s \sin m\phi, \\ \delta V_m &= \delta \left( \frac{\dot{v}_s}{c} \right)_m^c \cos m\phi + \delta \left( \frac{\dot{v}_s}{c} \right)_m^s \sin m\phi, \\ \delta Y_m &= \delta \tau_{Lm}^c \cos m\phi + \delta \tau_{Lm}^s \sin m\phi,\end{aligned}\quad (24)$$

where

$$\begin{aligned}\delta \left( \frac{\dot{u}_s}{c} \right)_m^c &= 2k^2 \left[ \frac{\bar{A}_m^o}{r_\alpha} - ik \bar{E}_m^e \right], \\ \delta \left( \frac{\dot{u}_s}{c} \right)_m^s &= 2k^2 \left[ \frac{\bar{B}_m^o}{r_\alpha} - ik \bar{F}_m^e \right], \\ \delta \left( \frac{\dot{w}_s}{c} \right)_m^c &= 2k^2 \left[ \bar{A}_m^e + ik \frac{\bar{E}_m^o}{r_\beta} \right],\end{aligned}$$

$$\begin{aligned}
\delta\left(\frac{\dot{w}_s}{c}\right)_m &= 2k^2\left[\bar{B}_m^e + ik\frac{\bar{F}_m^o}{r_\beta}\right], \\
\delta\sigma_m^c &= 2\rho c^2k^2\left[(\gamma-1)\frac{\bar{A}_m^o}{r_\alpha} - ik\gamma\bar{E}_m^e\right], \\
\delta\sigma_m^s &= 2\rho c^2k^2\left[(\gamma-1)\frac{\bar{B}_m^o}{r_\alpha} - ik\gamma\bar{F}_m^e\right], \\
\delta\tau_{Rm}^c &= 2\rho c^2k^2\left[-\gamma\bar{A}_m^e - ik(\gamma-1)\frac{\bar{E}_m^o}{r_\beta}\right], \\
\delta\tau_{Rm}^s &= 2\rho c^2k^2\left[-\gamma\bar{B}_m^e - ik(\gamma-1)\frac{\bar{F}_m^o}{r_\beta}\right], \\
\delta\left(\frac{\dot{v}_s}{c}\right)_m^c &= i2k_\beta^2\frac{\bar{C}_m^{(3)o}}{r_\beta}, \\
\delta\left(\frac{\dot{v}_s}{c}\right)_m^s &= i2k_\beta^2\frac{\bar{D}_m^{(3)o}}{r_\beta}, \\
\delta\tau_{Lm}^c &= -i2k_\beta^2\mu\bar{C}_m^{(3)e}, \\
\delta\tau_{Lm}^s &= -i2k_\beta^2\mu\bar{D}_m^{(3)e}.
\end{aligned} \tag{25}$$

The quantities  $r_\alpha$ ,  $r_\beta$ ,  $\rho$ ,  $\mu$ ,  $\gamma$ ,  $k_\beta$  refer to the layer in which the source occurs, with  $\rho$  and  $\mu$  being the density and shear modulus. The coefficients  $\bar{A}_m$ ,  $\bar{B}_m$ , etc. are given by (14) with the following modification. The series are separated into two parts;

$$\bar{A}_m = \bar{A}_m^e + \bar{A}_m^o, \tag{26}$$

with the  $e$  superscript denoting a series made up of terms with  $m+l$  even and the  $o$  superscript denoting a similar series from terms with  $m+l$  odd.

Equations (23) may be viewed as a set of simultaneous linear algebraic equations and solved for  $\hat{\Delta}_n^{(m)}$ ,  $\hat{\omega}_n^{(m)}$ ,  $\hat{\epsilon}_n^{(m)}$  in terms of known quantities. The result is

$$\begin{aligned}
2\hat{\Delta}_n^{(m)} &= \{R_1(J_{11}^R + J_{21}^R) - R_2(J_{12}^R + J_{22}^R) + J_{13}^R + J_{23}^R\}Y_m \\
&\quad + \{R_3(J_{11}^R + J_{21}^R) - R_4(J_{12}^R + J_{22}^R) + J_{14}^R + J_{24}^R\}Z_m, \\
2\hat{\omega}_n^{(m)} &= \{R_1(J_{31}^R + J_{41}^R) - R_2(J_{32}^R + J_{42}^R) + J_{33}^R + J_{43}^R\}Y_m \\
&\quad + \{R_3(J_{31}^R + J_{41}^R) - R_4(J_{32}^R + J_{42}^R) + J_{34}^R + J_{44}^R\}Z_m, \\
\hat{\epsilon}_n^{(m)} &= L_1X_m,
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
F_R R_1 &= (J_{13}^R - J_{23}^R)(J_{32}^R - J_{42}^R) - (J_{12}^R - J_{22}^R)(J_{33}^R - J_{43}^R), \\
F_R R_2 &= (J_{13}^R - J_{23}^R)(J_{31}^R - J_{41}^R) - (J_{11}^R - J_{21}^R)(J_{33}^R - J_{43}^R), \\
F_R R_3 &= (J_{14}^R - J_{24}^R)(J_{32}^R - J_{42}^R) - (J_{12}^R - J_{22}^R)(J_{34}^R - J_{44}^R), \\
F_R R_4 &= (J_{14}^R - J_{24}^R)(J_{31}^R - J_{41}^R) - (J_{11}^R - J_{21}^R)(J_{34}^R - J_{44}^R), \\
F_R &= (J_{12}^R - J_{22}^R)(J_{31}^R - J_{41}^R) - (J_{11}^R - J_{21}^R)(J_{32}^R - J_{42}^R), \\
L_1 &= (J_{11}^L J_{22}^L - J_{12}^L J_{21}^L)/(J_{11}^L - J_{21}^L),
\end{aligned}$$

and

$$\begin{aligned} Y_m &= (A_{R_{S1}})_{42} \delta U_m - (A_{R_{S1}})_{32} \delta W_m + (A_{R_{S1}})_{22} \delta Z_m - (A_{R_{S1}})_{12} \delta X_m, \\ Z_m &= -(A_{R_{S1}})_{41} \delta U_m + (A_{R_{S1}})_{31} \delta W_m - (A_{R_{S1}})_{21} \delta Z_m + (A_{R_{S1}})_{11} \delta X_m, \\ X_m &= -(A_{L_{S1}})_{21} \delta V_m + (A_{L_{S1}})_{11} \delta Y_m. \end{aligned} \quad (28)$$

Equations (27) give  $\hat{\Delta}_n^{(m)}$ ,  $\hat{\omega}_n^{(m)}$ ,  $\hat{e}_n^{(m)}$  in terms of the multipole coefficients specifying the source and the Haskell-Thomson layer matrices. Then (22) together with (12) give closed-form solutions for the displacements in the  $n$ th layer or half-space. In the following section we evaluate the integrals in (22) to extract the solutions for the steeply emergent body waves of primary interest here.

### COMPUTATION OF BODY WAVES

To compute the displacement potentials from (22), it is necessary to evaluate integrals of the form

$$\begin{aligned} I_m &= \int_0^\infty f_m(k, \omega) \exp(-ikr_\gamma \bar{z}) J_m(kr) dk, \\ \gamma &= \alpha_n \text{ or } \beta_n, m = 0, 1, 2, \dots \end{aligned} \quad (29)$$

For  $m = 0$  the dilatational potential,  $\bar{\Phi}_n$ , was evaluated at large distances from the source by Fuchs (1966) using saddle point methods. Hudson (1969a, b) encountered integrals very similar to those in (22) when solving for the body waves due to a point source of general form in a layered medium. As mentioned in the previous section, Hudson's solution is analogous to that obtained here, with the difference being in the specification of the source.

Hudson solved for the far-field body waves given by integrals like (29) by using contour integration in the complex plane and approximating the branch line contribution using the saddle point method. The details of this integration may be found in the works of Fuchs and Hudson and will not be reproduced here. It will suffice to give the results which are

$$I_m = i^{m+1} f_m(k, \omega) r_\gamma \exp(-ik_\gamma R)/R \quad (30)$$

where  $R^2 = r^2 + \bar{z}^2$ . The geometry is shown in Figure 2. The saddle point approximation is valid as long as  $R \gg r$ ; that is, as long as the receiver is sufficiently far from the base of the stack of plane layers. Alternatively, since  $r/R = \sin \theta_n$ , this approximation is valid for steeply emergent body waves.

With the solution (30), we may now write the far-field body-wave contribution to the potentials (22)

$$\begin{aligned} \bar{\Phi}_n(r, \phi, z) &= \sum_{m=0}^l (-1/k_{\alpha_n}^2) i^{m+1} r_{\alpha_n} \hat{\Delta}_n^{(m)} \exp(-ik_{\alpha_n} R)/R, \\ \bar{\Psi}_n(r, \phi, z) &= \sum_{m=0}^l (2/k_{\beta_n}^2) (i^m/k) r_{\beta_n} \hat{\omega}_n^{(m)} \exp(-ik_{\beta_n} R)/R, \\ \bar{\Omega}_n(r, \phi, z) &= \sum_{m=0}^l (i^{m+1}/k) r_{\beta_n} \hat{e}_n^{(m)} \exp(-ik_{\beta_n} R)/R. \end{aligned} \quad (31)$$

The cylindrical displacements  $(\bar{q}_n, \bar{v}_n, \bar{w}_n)$  then result from substituting (31) into (12). It is more convenient to deal with the displacement components in spherical coordinates where they are identified as the  $P$ ,  $SV$  and  $SH$  components of the propagating wave. In

the far-field, these components are given by

$$\begin{aligned}\bar{U}_P &= \sin \theta_n \frac{\partial \bar{\Phi}_n}{\partial r} + \cos \theta_n \frac{\partial \bar{\Phi}_n}{\partial z}, \\ \bar{U}_{SV} &= \cos \theta_n \frac{\partial^2 \bar{\Psi}_n}{\partial r \partial z} - \sin \theta_n k^2 \bar{\Psi}_n, \\ \bar{U}_{SH} &= -\frac{\partial \bar{\Omega}_n}{\partial r}.\end{aligned}\quad (32)$$

Then carrying out the indicated differentiations and retaining only terms of  $O(R^{-1})$ , the displacements are

$$\begin{aligned}\bar{U}_P &= \sum_{m=0}^l \left( -\frac{1}{k_{\alpha n}} \right) i^m r_{\alpha n} \hat{\Delta}_n^{(m)} \exp(-ik_{\alpha n} R)/R, \\ \bar{U}_{SV} &= \sum_{m=0}^l \left( \frac{2}{k_{\beta n}} \right) i^{m-1} r_{\beta n} \hat{\omega}_n^{(m)} \exp(-ik_{\beta n} R)/R, \\ \bar{U}_{SH} &= \sum_{m=0}^l i^m r_{\beta n} \hat{\epsilon}_n^{(m)} \exp(-ik_{\beta n} R)/R.\end{aligned}\quad (33)$$

These are the far-field body waves propagating into the underlying half-space at specified horizontal phase velocity  $c$ . Equations (33), together with (27), provide a straightforward computational algorithm for computing these displacements. A summary of the computational procedure is given below.

Assume that we have a seismic source specified in terms of multipole coefficients. We then wish to compute the  $P$  and  $S$  waves propagating into the half-space at takeoff angle  $\theta_n$ , in which case a different horizontal wave speed is required for the  $P$  and  $S$  computations. Alternatively, we wish to compute the displacements propagating at a fixed phase velocity,  $c$ .

Having the multipole coefficients,  $c$  and the azimuth  $\phi$ , the following steps are carried out for each frequency,  $\omega$ .

1. Compute the source layer potentials  $\bar{\phi}_s, \bar{\psi}_s, \bar{\Omega}_s$  from (13-16).
2. Compute the displacement-stress discontinuity quantities  $\delta U_m, \delta W_m$ , etc., from (24-25) for each  $m$ .
3. Compute the layer product matrices  $A_{R_{S1}}, A_{L_{S1}}, J^R, J^L$ . The latter two matrices are independent of the source, while the first two depend only on source depth.
4. Compute the source terms  $Y_m, Z_m, X_m$  from (29).
5. Compute  $\hat{\Delta}_n^{(m)}, \hat{\omega}_n^{(m)}, \hat{\epsilon}_n^{(m)}$  from (27).
6. Compute the displacement spectra at a selected distance  $R$  from (33).

Formulation of the solution in terms of Haskell-Thomson matrices is convenient for various modifications that may be of interest. For example, Dorman (1962) shows how to modify the layer matrices to account for the presence of a fluid layer and this modification can easily be carried through the algebra leading to (23) and (27).

#### COMPUTATION OF THEORETICAL SEISMOGRAMS

Our objective is to develop improved methods for computing theoretical seismograms at large (say greater than 1500 km) distances. Equations (33) give the  $P, SV, SH$  waves emanating into a homogeneous half-space underlying a stack of plane layers in which a seismic source is embedded. If this plane layered model is used to represent the crust in

the source vicinity, (33) represents the waves propagated into the upper mantle. The major restriction on the use of (33) is to situations for which the saddle point approximation is valid; that is, to small angles of emergence.

To compute theoretical seismograms we need to combine (33) with other methods for computing the remainder of the travel path to the receiver. There are a number of computational schemes that could be chosen but we have found it convenient to use generalized ray theory (Wiggins and Helmberger, 1974) for the upper mantle and Haskell-Thomson matrices (Haskell, 1962) for the crust in the vicinity of the receiver. Thus we break the travel path into three pieces:

1. The crust at the source down to a depth  $D$ .
2. The crust at the receiver to the same depth  $D$ .
3. A laterally homogeneous upper mantle extending from  $D$  to depths greater than the deepest turning point of interest.

The travel-path segments are linked by requiring that the velocities at  $D$  be the same in all three structures. This scheme gives a great deal of computational flexibility in that portions of the travel path can be varied without repeating the entire calculation. The actual implementation is described in a companion paper (Bache and Archambeau, 1976) where a number of examples are presented.

Methods similar to those outlined here have been used by a number of investigators. Most closely related is the work by Douglas and colleagues (e.g., Douglas *et al.*, 1972, 1974; Cullen and Douglas, 1975) which uses the method of Hudson (1969a, b) for embedding a seismic source in a plane layered model of the source crustal structure. Also, these authors represent the geometric spreading effect of the upper mantle by a constant which is a function only of epicentral distance (Carpenter, 1966) rather than the detailed generalized ray theory method we prefer. We note that beyond the triplications the upper mantle effect does essentially reduce to a distance-dependent constant, although its value depends on the earth model used. Other authors who have used similar techniques include Kogus (1968) and Hasegawa (1972, 1973) who used Fuchs (1966) formulation for the source crustal transfer function and Julian and Anderson's (1968) geometric spreading factor.

For simple point sources (center of dilatation, couple or double couple) generalized ray theory can be directly applied to compute theoretical seismograms (e.g., Wiggins and Helmberger, 1974; Müller, 1971). Another successful theoretical seismogram computing technique is the reflectivity method (Fuchs, 1975; Fuchs and Müller, 1971; Kind and Müller, 1975) although it too is restricted to elementary source representations.

The major difference between the method presented in this paper and the others mentioned above is in the source representation. Our method is essentially independent of the complexity of the source as long as the outgoing elastic wave field can be expanded in spherical harmonics.

#### ACKNOWLEDGMENTS

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Air Force Office of Scientific Research under Contract F44620-C-0063 with Systems, Science and Software, and Contract F44620-72-C-0078 with the California Institute of Technology.

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Manuscript received April 9, 1976